اللوغارتم الطبيعي

اولا: Ln

- مشتقة ln

$$f(x) = ln\ (\text{دالة}) \quad \rightarrow f'(x) = \frac{\text{مشتقة الداله}}{\text{الداله نفسها}}$$

سؤال: اذا كان $f(x) = ln\left|x^5 + 2\right|$ فأوجد $f'(x)$

Sol: $f'(x) = \frac{5x}{x^5+2}$

سؤال: اذا كان $f(x) = ln\left|cosx\right|$ فأوجد $f'(x)$

Sol: $f'(x) = \frac{-sinx}{cosx}$

سؤال: اذا كان $f(x) = ln\left|sin4x\right|$ فأوجد $f'(x)$

Sol: $f'(x) = \frac{4cos4x}{sin4x}$

- **ثوابت حفظ**

$ln\ 1 = 0$,

$ln\ e = 1$

$ln\ xy = ln\ x + ln\ y$,

$ln\ \frac{x}{y} = ln\ x - ln y$

$ln\ x^a = a\ ln\ x$

$ln\ e^x = x$

- تكامل ln : توفير مشتقة المقام في البسط (بشرط ان يكون المقام مرفوع الى اس 1)

$$\int \frac{\text{مشتقه الدالة}}{(\ \text{دالة}\ )^1}\ dx = ln\ |\text{الدالة}\ | + C$$

Ex: $\int \frac{2X+1}{x^2+x}\ dx$

$= ln|x^2 + x| + C$

Ex: $\int \frac{3x^2+4}{x^3+4x+1}\ dx$

$= ln|x^3 + 4x + 1| + C$

Ex: $\int \frac{x^4+1}{x^5+5x}\ dx$

$= \frac{1}{5}\int \frac{5(x^4+1)}{x^5+5x}\ dx$

$= \frac{1}{5} ln|x^5 + 5x| + C$

Ex: $\int (x+2)^{-1}\ dx$

$= \int \frac{1}{(x+2)}\ dx$

$= ln|x + 2| + C$

Ex: $\int \frac{1}{x^9+x}\, dx$

$= \int \frac{1}{x^9\,(1+x^{-8})}\, dx$

$= \int \frac{x^{-9}}{(1+x^{-8})}\, dx$

$= \frac{-1}{8} \int \frac{-8\,x^{-9}}{(1+x^{-8})}\, dx$

$= \frac{-1}{8} \int \frac{-8\,x^{-9}}{(1+x^{-8})}\, dx$

$= \frac{-1}{8} \ln \left| (1+x^{-8}) \right| + c$

Ex: $\int \frac{x}{x^2+x^4}\, dx$

$= \int \frac{x}{x^4\,(x^{-2}+1)}\, dx$

$= \int \frac{x^{-3}}{(x^{-2}+1)}\, dx$

$= \frac{-1}{2} \int \frac{-2x^{-3}}{(x^{-2}+1)}\, dx$

$= \frac{-1}{2} \ln \left| (x^{-2}+1) \right| + c$

Ex: $\int \frac{sec^2x}{5-tanx}\ dx$ sec^2x مشتقة المقام

$= -\ln|5+\tan x| + C$

Ex: $\int \frac{\cos\theta}{1+sin\theta}\ d\theta$

$= ln\ |1+sin\theta|\ +\ c$

Ex: $\int \frac{x+cos2x}{x^2+2x}\ dx$ $2x+2cos2x$ مشتقة المقام

$= \frac{1}{2}\int \frac{2(x+cos2x)}{x^2+sin2x}\ dx$

$= \frac{1}{2}\ ln\ |x^2+sin2x|\ +\ c$

Ex: $\int cot^3\ 5x\ dx$

$= \int cot5x\ cot^2\ 5x\ dx\ = \int cot5x\ (csc^2\ 5x-1)\ dx$

$= \int (cot5x\ csc^2\ 5x - cot5x)\ dx\ = \int cot5x\ csc^2\ 5x\ dx\ - \int \frac{cos5x}{sin5x}\ dx$

$= -\frac{1}{5}\int cot5x(-5csc^2\ 5x)\ dx\ -\frac{1}{5}\int \frac{5cos5x}{sin5x}\ dx$

$= -\frac{1}{5}\ .\frac{cot^2 5x}{2}\ -\frac{1}{5}\ ln\ sin5x\ +\ c$

$= -\frac{1}{10}\ cot^2 5x\ -\frac{1}{5}\ ln\ sin5x\ +\ c$

ثانيا: e^n

- مشتقه e^x :

$$f(x) = e^u \quad \rightarrow \quad f'(x) = u' e^u$$

Ex: $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

Ex: $f(x) = e^{x^2}$

$f'(x) = 2x\, e^{x^2}$

Ex: $f(x) = e^{\tan x}$

$f'(x) = e^{\tan x} \cdot \sec^2 x$

Ex: $f(x) = e^{2x+1}$

$f'(x) = 2\, e^{2x+1}$

Ex: $f(x) = e^{(5x+1)^2}$

$f'(x) = 2(5x+1) * 5\ e^{(5x+1)^2}$

$f'(x) = 10(5x+1)\, e^{(5x+1)^2}$

$f'(x) = (50x+10)\, e^{(5x+1)^2}$

- $e^{\ln x} = x$
- $e^0 = 1$

ثوابت حفظ

- تكامل e^x : يجب توفير مشتقة الاس وبعد توفيرها تحذف فيكون التكامل الدالة نفسها.

$$\int u' e^u \, du = e^u + c$$

Ex: $\int 2xe^{x^2} \, dx$

$= e^{x^2} + c$

Ex: $\int xe^{x^2} \, dx$

$= \frac{1}{2} \int 2xe^{x^2} dx$

$= \frac{1}{2} e^{x^2} + c$

Ex: $\int (1 + e^x)^2 e^x \, dx$

$= \frac{(1+e^x)^3}{3} + c$

Ex: $\int cosx \; e^{sinx} \, dx$

$= e^{sinx} + c$

Ex: $\int sec^2 2x \, e^{tan2x} \, dx$

$= \frac{1}{2} \int 2sec^2 2x \, e^{tan2x} \, dx$

$= \frac{1}{2} e^{tan2x} + c$

ثالثا: a^u :

المشتقة: $f'(x) = a^u \cdot u' \cdot \ln a$

Ex: $f(x) = 3^{2x-5}$

$$f'(x) = 3^{2x-5} \cdot (2) \cdot \ln 3$$

$$= (2 \ln 3) \cdot 3^{2x-5}$$

Ex: $f(x) = 9^{\sqrt{x}}$

$$f'(x) = 9^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \cdot \ln 9$$

$$f'(x) = \frac{\ln 9}{2\sqrt{x}} 9^{\sqrt{x}}$$

Ex: $f(x) = 7^{\frac{-x}{4}}$

$$f'(x) = 7^{\frac{-x}{4}} \cdot (\frac{-1}{4}) \cdot \ln 7$$

Ex: $f(x) = 5^{\sin x}$

$$f'(x) = 5^{\sin x} \cdot \cos x \cdot \ln 5$$

Ex: $f(x) = 2^{\tan x}$

$$f'(x) = 2^{\tan x} \cdot \sec^2 x \cdot \ln 2$$

التكامل: $\int u' a^u \, du = \frac{a^u}{\ln a} + c$

Ex: $\int 2\,(8^{2x})\, dx$

$= \frac{8^{2x}}{ln8} + c$

Ex: $\int\ (5^{-3x})\, dx$

$= \frac{-1}{3}\int -3\,(5^{-3x})\, dx$

$= \frac{-1}{3} * \frac{5^{3x}}{ln5} + c$

Ex: $\int x\ (6^{2x^2})\, dx$

$= \frac{1}{4}\int 4x\,(6^{2x^2})\, dx$

$= \frac{1}{4} * \frac{6^{2x^2}}{ln6} + c$

Ex: $\int cosx\ (9^{\,sinx})\, dx$

$= \frac{9^{\,sinx}}{ln9} + c$