مشتقة الدوال المثلثية:

$f(x) = \sin x \rightarrow f(x)' =$ مشتقة الزاوية $\cos x$

$f(x) = \cos x \rightarrow f(x)' = -$ مشتقة الزاوية $\sin x$

$f(x) = \tan x \rightarrow f(x)' =$ مشتقة الزاوية $\sec^2 x$

$f(x) = \cot x \rightarrow f(x)' = -$ مشتقة الزاوية $\csc^2 x$

$f(x) = \sec x \rightarrow f(x)' =$ مشتقة الزاوية $\sec x \tan x$

$f(x) = \text{scs } x \rightarrow f(x)' = -$ مشتقة الزاوية $\csc x \cot x$

تكامل الدوال المثلثية

ملاحظة : نوفر مشتقة الزاوية ونكامل.

$\int$ u مشتقة $\sin u\, dx = -\cos u + c$

$\int$ u مشتقة $\cos u\, dx = \sin u + c$

$\int$ u مشتقة $\sec^2 u\, dx = \tan u + c$

$\int$ u مشتقة $\csc^2 u\, dx = -\cot u + c$

$\int$ u مشتقة $\sec u \tan u\, dx = \sec u + c$

$\int$ u مشتقة $\csc u \cot u\, dx = -\csc u + c$

$\int$ u مشتقة $\tan u\, dx = \ln|\sec u| + c$

$\int$ u مشتقة $\cot u\, dx = -\ln|\csc u| + c$

مثال : جد تكاملات الاتية:

1) $\int (\sin 4x\, dx$

$$= \frac{1}{4}\int 4(sin4x\, dx = \frac{-1}{4}\cos 4x + c$$

2) $\int \cos 2x\, dx$

$$= \frac{1}{2}\int 2\cos 2x\, dx = \frac{1}{2}\sin 2x + c$$

3) $\int \sec^2 3x\, dx$

$$= \frac{1}{3}\int 3\sec^2 3x\, dx = \frac{1}{3}\tan 3x + c$$

4) $\int \csc^2 x\, dx$

$$= -\cot x + c$$

5) $\int \sec 5x \tan 5x\, dx$

$$= \frac{1}{5}\int 5\sec 5x \tan 5x\, dx = \frac{1}{3}\sec 5x + c$$

6) $\int \csc x \cot x\, dx$

$$= -\csc x + c$$

7- $\int 5 \tan 5x \, dx$

$= \ln \left| \sec 5x \right| + c$

8- $\int \cot 2x \, dx$

$= -\frac{1}{2} \ln \left| \csc u \right| + c$

9- $\int - x \sin x^2 \, dx$

$= \frac{-1}{2} \int 2x \sin x^2 \, dx = \frac{1}{2} \cos 2x^2 + c$

10- $\int x^3 \cos 2x^4 \, dx$

$= \frac{1}{8} \int 8x^3 \cos 2x^4 \, dx = \frac{1}{8} \sin 2x^4 \, dx + c$

11- $\int x \sec 5x^2 \tan 5x^2 \, dx$

$= \frac{1}{10} \int 10 \, x \sec 5x^2 \tan 5x^2 \, dx = \frac{1}{10} \sec 5x^2 + c$

12- $\int x^2 \sin x^3 \, dx$

$= \frac{1}{3} \int 3x^2 \sin x^3 \, dx = \frac{-1}{3} \cos x^3 + c$

13- $\int(\cos 2x + \sin 2x)\, dx$

$$= \frac{1}{2}\sin 2x + \left(\frac{-1}{2}\ cos\ 2x\right) + c$$

$$= \frac{1}{2}\ sin\ 2x\ - \frac{1}{2}\ cos\ 2x\ + c$$

14- $\int(\sin x\ - \cos x)^7\ (\ cos\ x\ +\ sin\ x)\, dx$

$$= \frac{(sin\, x - cos\, x)^8}{8} + c$$

$$= \frac{1}{8}(sin\, x - cos\, x)^8 + c$$

15- $\int \frac{sec\sqrt{x-1}\ tan\sqrt{x-1}}{\sqrt{x-1}}\, dx$

$$= \int (x-1)^{-1/2}\ sec\sqrt{x-1}\ tan\sqrt{x-1} dx$$

$$= 2\int \frac{1}{2}(x-1)^{-1/2}\ sec\sqrt{x-1}\ tan\sqrt{x-1} dx$$

$$= 2\ sec\sqrt{x-1} + c$$

ملاحظة : في دالة الدالة اذا كان ($cscx$ او $secx$) مرفوع الى اس غير (2) نجزء الاس الى (1 والباقي)

16- $\int \tan x\ sec^{-8}x\ dx$

$$= \int \tan x\ secx\ \ sec^{-9}x\ dx$$

$$= \frac{-1}{8}sec^{-8}x + c = -\frac{1}{8}sec^{-8}x + c$$

17- $\int \cos x \sin^3 x \, dx$

$= \frac{1}{4}\sin^4 x + c$

18- $\int \sin 2x \cos^{-5} 2x \, dx$

$= \frac{-1}{2}\int -2 \sin 2x \cos^{-5} 2x \, dx$

$= \frac{1}{8}\cos^{-4} x + c$

19- $\int \csc x \cot x \csc^4 x \, dx$

$= \frac{1}{5}\csc^5 x + c$

20 - $\int \sin x \sin^3 (\cos x) \, dx$

$= -\int - \sin x \sin^3 (\cos x) \, dx$

$= -\frac{1}{4}\sin^4 (\cos x) \, dx$

واجبات:

1- $\int 9 \sin 3x \ dx$

2- $\int x \tan (2x2 + 1) \, dx$

3- $\int \cot (7 - \frac{x}{2}) \, dx$

4- $\int \sin (3x + 2) + \cos(2-3x) \, dx$

5- $\int \sec^2 (4x) \, dx$

6- $\int x^2 \csc (3 - \frac{x^2}{3}) \, dx$

7- $\int x^2 \csc (2x^3) \cot(2x^3) \, dx$

8- $\int \cos 6x \cos(9 + 4\sin 6x) \, dx$

9- $\int \cos^3 x \sin x \, dx$

10- $\int \frac{1}{\sqrt{x}} \sin \sqrt{x} \, dx$

11- $\int (1 + \sin t)^2 \cos t \, dt$

12- $\int x\cos(3x^2) \, dx$

13- $\int x^2 \sec^2 x^3 dx$

14- $\int \cos^3 2t \sin 2t \, dt$

15- $\int \cos x \sqrt{2 - sin4x} \ dx$

16- $\int \tan^3 5x \sec^2 5x \, dx$

17- $\int \sin 3x \sec^2 \cos 3x \, dx$

18- $\int (1-\cos x)^3 \sin x dx$

19- $\int (1- \sin 2x)^{1/3} \cos 2x dx$

20- $\int x^7 \tan (8x^8 + 6) \, dx$

21- $\int \sin (7- \cos 3x) \sin 3x \, dx$

22- $\int \frac{1}{\sqrt{t}} \cos \sqrt{t} \, dt$

23- $\int \frac{\sec x \tan x}{(3+2secx)^2} \ dx$

24- $\int \frac{(\cos x - \sin x)}{(sinx+cosx)^3} \ dx$

25- $\int (2+ \tan^2 x) \sec^2 x \, dx$