- **المتطابقات ***

اولا: المتطابقة الذهبية : (شرط تشابه الزوايا):

$$sin^2x + cos^2x = 1$$

من هذه المتطابقة نجد:

$$\mathbf{sin^2x = 1 - cos^2x}$$

$$\mathbf{cos^2x = 1 - sin^2x}$$

سؤال: جد التكاملات الاتية:

Ex. $\int (sin^2x + cos^2x)\ dx$

$$= \int 1\ dx = x + c$$

Ex. $\int \sqrt{1 - sin^2x}\ dx$

$$= \int \sqrt{cos^2x}\ dx$$
$$= \int cosx\ dx = sinx + c$$

Ex. $\int cos^3 x\ dx$

$$= \int cos^2 x\ cosx\ dx$$

$$= \int (1 - sinx)cosx\ dx$$

$$= \int cosx - sin^2x\ cosx\ dx$$

$$= sinx - \frac{sin^3x}{3} + c$$

$$= sinx - \frac{1}{3} sin^3x + c$$

Ex. $\int sin^3 x\, dx$

$= \int \left(sin^2 x\right) \sin x\, dx$

$= \int (1 - cos^2 x)\, sinx\ dx$

$= \int (sinx - sinx\, cos^2 x\ dx$

$= -cosx - \frac{cos^3 x}{3} + c$

$= -cos\, x - \frac{1}{3} cos^3 x + c$

Ex. $\int sin^5 x\, dx$

$= \int \left(sin^2 x\right)^2 \sin x\, dx$

$= \int (1 - cos^2 x)^2\, sinx\ dx$

$= \int (1 - 2cosx + cos^2 x)\, sinx\ dx$

$= \int (sinx - 2cosx\, sinx + cos^2 x\, sinx)\, dx$

$= -cosx + 2\, \frac{cos^2 x}{2} - \frac{cos^3 x}{3} + c$

$= -cosx + cos^2 x - \frac{cos^3 x}{3} + c$

ملاحظة : من المتطابقة الذهبية يمكن ان نستخدم طريقة الفرق بين مربعين وكما يلي :

$$\boldsymbol{sin^2x = 1 - cos^2x = (1 - cos\,x)(1 + cos\,x)}$$

$$\boldsymbol{cos^2x = 1 - sin^2x = (1 - sin\,x)(1 + sin\,x)}$$

Ex. $\int \frac{sin^2x}{1-cos\,x}\,dx$

$$= \int \frac{1 - \cos^2 x}{1 - \cos x}\,dx$$

$$= \int \frac{(1 - \cos x)(1 + \cos x)}{1 - cos\,x}\,dx$$

$$= \int (1 + cos\,x)\,dx = x + sinx + c$$

Ex. $\int \frac{cos^2x}{1+sin\,x}\,dx$

$$= \int \frac{1 - sin^2 x}{1 - \sin x}\,dx$$

$$= \int \frac{(1 - \sin x)(1 + \sin x)}{1 - sin\,x}\,dx$$

$$= \int (1 - sin\,x)\,dx = x + cosx + c$$

ثانيا: متطابقات نصف الزاوية :

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

سؤال جد التكاملات الاتية:

$$Ex. \int \frac{\cos 4x}{\cos 2x - \sin 2x}\, dx$$

$$= \int \frac{\cos^2 2x - \sin^2 2x}{\cos 2x - \sin 2x}\, dx$$

$$= \int \frac{(\cos 2x - \sin 2x)(\cos 2x + \sin 2x)}{\cos 2x - \sin 2x}\, dx$$

$$= \int (\cos 2x + \sin 2x)\, dx$$

$$= \frac{1}{2}\sin 2x - \frac{1}{2}\cos 2x + c$$

$$Ex. \int \sin 6x \ \cos^2 3x\, dx$$

$$= \int 2 \sin 3x \cos 3x \cos^2 3x\, dx$$

$$= 2\int \sin 3x \ \cos^3 3x\, dx$$

$$= 1 * \frac{1}{-3} \int -3\sin 3x \ \cos^3 3x\, dx$$

$$= \frac{-2}{3} \cdot \frac{\cos^4 3x}{4} + c$$

$$= \frac{-1}{6} \cos^4 3x + c$$

Ex. $\int (\cos^2 x - \sin^2 x)\, dx$

$$= \int \cos 2x\, dx$$

$$= \frac{1}{2}\sin 2x + c$$

Ex. $\int \sin^3 x \cos^3 x\, dx$

$$= \int (\sin x \cos x)^3\, dx$$

$$= \int \left(\frac{\sin 2x}{2}\right)^3 dx$$

$$= \int \frac{\sin^3 2x}{(2)^3} dx$$

$$= \frac{1}{8}\int \sin^3 2x\ dx$$

$$= \frac{1}{8}\int \sin^2 2x \sin 2x\, dx$$

$$= \frac{1}{8}\int (1 - \cos^2 2x) \sin 2x\, dx$$

$$= \frac{1}{8}\int (\sin 2x - \cos^2 2x \sin 2x)\ dx$$

$$= \frac{1}{8}\left[\int \sin 2x\, dx - \frac{1}{-2}\int -2\cos^2 2x \sin 2x\, dx\right]$$

$$= \frac{1}{8}\left[\frac{-1}{2}\cos 2x + \frac{1}{2}.\frac{\cos^3 2x}{3}\right] + c$$

$$= -\frac{1}{16}\cos 2x + \frac{1}{48}\cos^3 2x + c$$

من المطابقة $cos\ 2x = cos^2x - sin^2x$ وتعوض المطابقة الذهبية

$cos^2x = 1 - sin^2x$

$sin^2x = 1 - cos^2x$

نعوض قيمة cos^2x و sin^2x في المطابقة $cos\ 2x = cos^2x - sin^2x$ نحصل على:

$$\boldsymbol{\cos 2x = 1 - 2\,sin^2x}$$

$$\boldsymbol{cos\ 2x = 2cos^2x - 1}$$

Ex. $\int (1 - 2\sin^2 x\ dx$

$$= \int cos\ 2x\ dx$$

$$= \frac{1}{2} sin\ 2x + c$$

$Ex. \int sin2x\ cos\ 4x\ dx$

$$= \int sin2x(2\,cos^2 2x - 1)\ dx$$

$$= \int (2sin2x\ cos^2 2x - sin2x)dx$$

$$= -\frac{cos^3 2x}{3} - \frac{1}{2} cos2x + c$$

$$= \frac{-1}{3} cos^3 2x - \frac{1}{2} cos2x + c$$

ثالثا: متطابقات ضعف الزاوية :

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

سؤال جد تكاملات الاتية:

Ex. $\int \cos^2 x\, dx$

$$= \frac{1}{2}\int (1 + \cos 2x)\, dx$$

$$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + c = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$$

Ex. $\int \sin^4 x\, dx$

$$= \int (\sin^2 x)^2\, dx = \int \left(\frac{1}{2}(1 - \cos 2x)\right)^2 dx = \int \frac{1}{4}(1 - \cos 2x)^2\, dx$$

$$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x)\right) dx$$

$$= \frac{1}{4}\left[x - 2 * \frac{1}{2}\sin 2x + \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right)\right] + c$$

$$= \frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x + c$$

$$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$$

$Ex.\ \int \sqrt{1-sin2x}\ dx$

$= \int \sqrt{sin^2x + cos^2x - 2sinx\ cosx}\ dx$

$= \int \sqrt{cos^2x - 2sinx\ cosx + sin^2x}\ dx$

$= \int \sqrt{(cosx - sinx)^2}\ dx$

$= \mp \int (\ cosx - sinx\ )\ dx$

$= \mp (\ sinx\ +\ cosx\ ) + c$

رابعا: متطابقات (tan^2 , cot^2) :

$$tan^2x = sec^2x - 1$$

$$cot^2x = csc^2x - 1$$

سؤال: جد تكاملات الاتية:

$$Ex. \int cot^2 5x\, dx$$

$$= \int (csc^2 5x - 1)\, dx$$

$$= \frac{-1}{5} cot5x - x + c$$

$$Ex. \int tan^2 4x\, dx$$

$$= \int sec^2 4x - 1\, dx$$

$$= \frac{1}{4} tan4x - x + c$$

خامسا: متطابقات (cot^n , tan^n , sec^n , csc^n)

$$\tan^n x = \frac{\sin^n x}{\cos^n x}$$

$$\cot^n x = \frac{\cos^n x}{\sin^n x} = \frac{1}{\tan^n x}$$

$$\sec^n x = \frac{1}{\cos^n x}$$

$$\csc^n x = \frac{1}{\sin^n x}$$

سؤال: جد تكاملات الاتية:

$EX. \int tanx\ cosx\ dx$

$$= \int \frac{\sin x}{cosx} . cosx\ dx$$

$$= \int sin\ x\ dx = -cosx + c$$

Ex. $\int \frac{1+\tan^2 x}{\tan^3 x} dx$

$$= \int tan^{-3} x\ (1 + tan^2 x)\ dx = \int tan^{-3} x\ sec^2 x\ dx$$

$$= \frac{tan^{-2} x}{-2} + c \qquad = \frac{-1}{2\ tanx} + c$$

Ex. $\int \frac{\tan x}{\cos^2 x} dx$

$$= \int \tan x\ sec^2 x\ dx = \frac{tan^2 x}{2} + c \qquad = \frac{1}{2} \tan^2 x + c$$