التكامل المحدد definite integral:

لتكن الدالة f(x) دالة مستمرة على المجال [a , b] ولتكن F(x) تكاملا غير محدد للدالة f(x) فان التكامل المحدود يكون بشكل الاتي:

$$\int_a^b f(x)\,dx = [\,F(x)\,]_a^b = F(b) - F(a)$$

خواص التكامل المحدد :

اولا: $\int_a^a f(x)\,dx = 0$

ثانيا: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

ثالثا: اذا كانت c ∋ [a , b] فان $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx = \int_c^b f(x)\,dx$

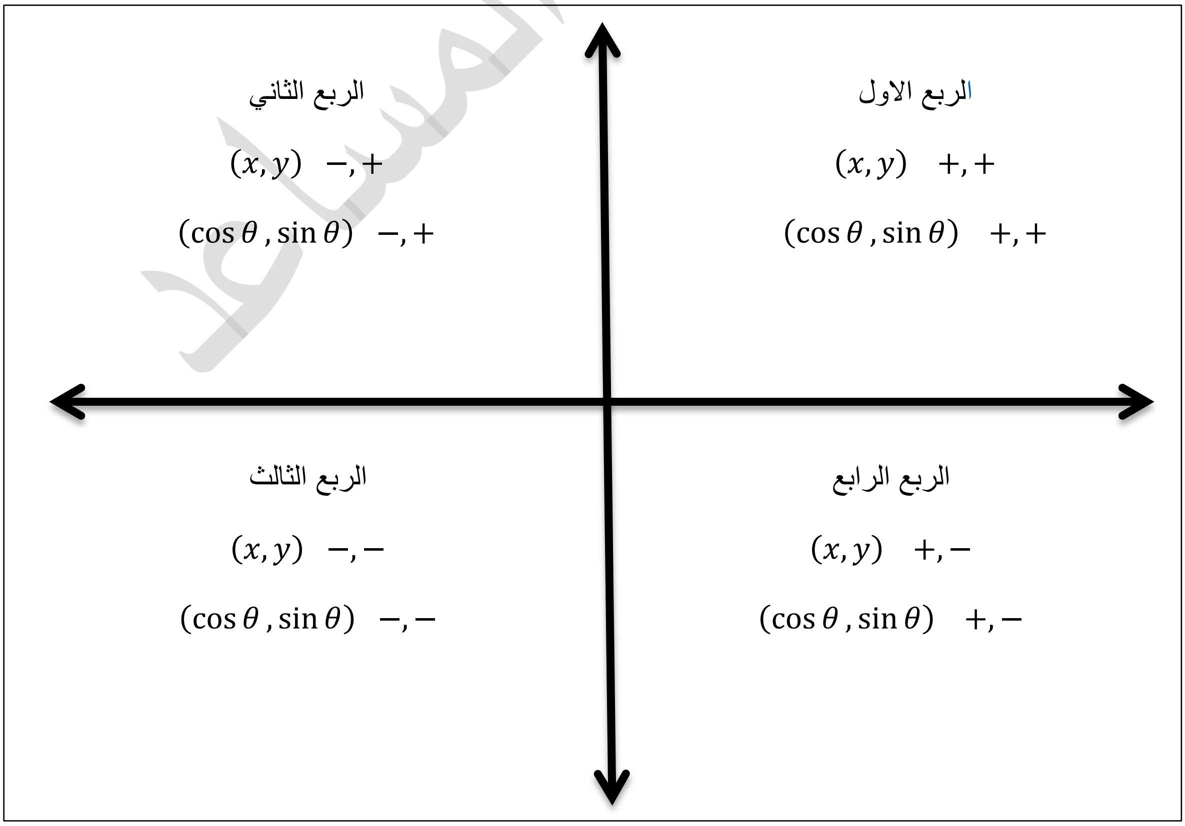
رابعا: $\int_a^a u\,f(x)\,dx = u\int_a^b f(x)\,dx$

خامسا: التكامل المحدد يوزع على الجمع والطرح:

$$\int_a^b f(x) \pm g(x)\ dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

جدول ايجاد قيم الزاوية للدوال المثلثية:

القياس الستيني	القياس الدائري	$\sin\theta$	$\cos\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60 عكس الزاوية 30	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90 عكس الزاوية 0	$\frac{\pi}{2}$	1	0	غير معرف
180 تشبه الزاوية 0 بعكس الاشارة	π	0	-1	0
270 نفس الزاوية 90 بعكس الاشارة	$\frac{3\pi}{2}$	-1	0	غير معرف
360 تشبه الزاوية 0	2π	0	1	0



Ex: $\int_0^1 2\, dx$

$=2\,[\,x]_0^1$

$=\ 2\ [1-0]\ = 2$

Ex: $\int_1^2 x\, dx$

$=\frac{1}{2}[\ x^2]_1^2$

$=\ \frac{1}{2}\ [2^2\ -\ 1^2]$

$=\ \frac{1}{2}\ [4\ -\ 1]$

$=\ \frac{1}{2}\ [3]=\frac{3}{2}$

Ex: $\int_0^3 x^3-4x+1\, dx$

$=\ [\ \frac{x^4}{4}-2x^2+x]_0^3$

$=\ [\ \frac{(3)^4}{4}-2(3)^2+(3)\ ]-\ [\ \frac{(0)^4}{4}+2(0)^2+(0)]$

$=[\frac{81}{4}$ - 18 + 3] – [0]

$=[\frac{81}{4}$ - 15] – [0]

$=\frac{81-60}{4}\ =\frac{21}{4}$

Ex: $\int_{\ln 3}^{\ln 5} e^{2x}\, dx$

$= \frac{1}{2}\int_{\ln 3}^{\ln 5} 2e^{2x} dx$

$= \frac{1}{2}\ [e^{2x}]_{\ln 3}^{\ln 5}$

$= \frac{1}{2}\ [e^{2ln5} - e^{2ln3}]$

$= \frac{1}{2}\ [e^{ln5^2} - e^{ln3^2}]$

$= \frac{1}{2}[\ 5^2 - 5^2\ ]$

$= \frac{1}{2}\ [25 - 9]$

$= \frac{1}{2}\ [16]$

$= 8$

Ex: $\int_{0}^{\ln 2} e^{-x} dx$

$= -\int_{0}^{\ln 2} -e^{-x} dx$

$= [-e^{-x}]_{0}^{ln 2}$

$= -[e^{-ln2} - e^{0}]$

$= -[e^{ln2^{-1}} - 1]$

$= -[2^{-1} - 1]$

$= -\left[\frac{1}{2} - 1\right] = -\frac{1}{2} + 1 = \frac{1}{2}$

Ex: $\int_{0}^{1} (1 + e^{x})^{2} e^{x} dx$

$= \left[\frac{(1+e^{x})^{3}}{3}\right]_{0}^{1}$

$= \frac{(1+e^{1})^{3}}{3} - \frac{(1+e^{0})^{3}}{3}$

$= \frac{(1+e)^{3}}{3} - \frac{(1+1)^{3}}{3}$

$= \frac{(1+e)^{3}}{3} - \frac{(2)^{3}}{3}$

$= \frac{(1+e)^{3}}{3} - \frac{8}{3}$

$= \frac{(1+e)^{3}-8}{3}$

Ex: $\int_1^4 \frac{e^{\sqrt{x}}\,dx}{2\sqrt{x}}$ مشتقة الاس $\frac{1}{2\sqrt{x}}$

$= [\, e^{\sqrt{x}} \,]_1^4$

$= e^{\sqrt{4}} - e^{\sqrt{1}}$

$= e^2 - e$

$= e\,(e - 1)$

سؤال: جد التكامل التالي: $\int_{-1}^{2} |x|\, dx$

الجواب:

$$|X| = \begin{cases} X, & \text{اذا كانت } X \geq 0 \\ -X, & \text{اذا كانت } X < 0 \end{cases}$$

$\int_{-1}^{2} |x|\, dx = \int_{-1}^{0} -x\, dx + \int_{0}^{2} x\, dx$

$= -\frac{1}{2}[\,X^2]_{-1}^{0} + \frac{1}{2}[\,X^2]_{0}^{2}$

$= -\frac{1}{2}\,[(0)^2 - (-1)^2] + \frac{1}{2}\,[(2)^2 - (0)^2]$

$= -\frac{1}{2}\,[-1] + \frac{1}{2}\,[4]$

$= \frac{1}{2} + 2 = \frac{5}{2}$

Ex: $\int_1^2 x\, e^{-\ln x}\, dx$

$$= \int_1^2 \frac{x}{e^{\ln x^1}}\, dx = \int_1^2 \frac{x}{x}\, dx = \int_1^2 dx$$

$$= [\,x\,]_1^2 = 2 - 1 = 1$$

طريقة ثانية

$$\int_1^2 x\, e^{\ln x^{-1}}\, dx = \int_1^2 x.x^{-1}\, dx = \int_1^2 dx$$

Ex: $\int_1^8 \frac{\sqrt{\sqrt[3]{x}-1}}{\sqrt[3]{x^2}}\, dx = 2$

مشتقة داخل القوس $\frac{1}{3\sqrt[3]{x^2}}$

$$\text{L.H} = \int_1^8 \frac{(\sqrt[3]{x}-1)^{\frac{1}{2}}}{\sqrt[3]{x^2}}\, dx$$

$$= 3\int_1^8 \frac{(\sqrt[3]{x}-1)^{\frac{1}{2}}}{3\sqrt[3]{x^2}}\, dx$$

$$= 3\left[\frac{(\sqrt[3]{x}-1)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^8 = 2\left[\sqrt{(\sqrt[3]{x}-1)^3}\right]_1^8$$

$$= 2\left[\sqrt{(\sqrt[3]{8}-1)^3} - \sqrt{(\sqrt[3]{1}-1)^3}\right]$$

$$= 2\left[\sqrt{(2-1)^3} - 0\right] = 2[\,1\,] = 2$$

$$\text{L.H} = \text{R.H}$$

$\cos -x = \cos x$
$\sin -x = -\sin x$
$\tan -x = -tanx$

Ex: $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin x \; dx$

$=[-cosx]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$

$= -\left[\cos\frac{\pi}{2} - \cos\frac{-\pi}{2}\right]$

$= -\left[\cos\frac{\pi}{2} - \cos\frac{\pi}{2}\right]$

$= -[0-0] = 0$

Ex: $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x \; dx$

$=[sinx]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$

$= \left[\sin\frac{\pi}{2} - \sin\frac{-\pi}{2}\right]$

$= \left[\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right]$

$= [1+1] = 2$

Ex: $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}\, dx$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{(\sin)^{\frac{1}{2}}}\, dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin)^{-\frac{1}{2}} \cos x\, dx$$

$$= \left[\frac{(\sin x)^{\frac{1}{2}}}{\frac{1}{2}}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 2\left[\sqrt{\sin\frac{\pi}{2}} - \sqrt{\sin\frac{\pi}{6}}\right]$$

$$= 2\left[\sqrt{1} - \sqrt{\frac{1}{2}}\right]$$

$$= 2\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= 2 - \frac{2}{\sqrt{2}}$$

$$= 2 - \sqrt{2}$$

$$= 2 - 1.4$$

$$= 0.6$$

Ex: $\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} cos^2 2x \ dx$

$= \int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}(1+\cos 4x) \ dx$

$= \frac{1}{2}\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} (1+\cos 4x) \ dx$

$= \frac{1}{2}[\ x + \frac{1}{4}\sin 4x\ ]_{\frac{-\pi}{3}}^{\frac{\pi}{3}}$

$= \frac{1}{2}[(\ \frac{\pi}{3} + \frac{1}{4}\sin 4\frac{\pi}{3}) - (\ \frac{-\pi}{3} + \frac{1}{4}\sin 4\frac{-\pi}{3})]$

$= \frac{1}{2}[(\ \frac{\pi}{3} + \frac{1}{4}\sin 4\frac{\pi}{3}) - (\ \frac{-\pi}{3} - \frac{1}{4}\sin 4\frac{\pi}{3})]$

$= \frac{1}{2}[(\ \frac{\pi}{3} + \frac{1}{4} * \frac{-\sqrt{3}}{2}) - (\frac{-\pi}{3} - \frac{1}{4} * \frac{-\sqrt{3}}{2})]$

$= \frac{1}{2}[(\ \frac{\pi}{3} - \frac{\sqrt{3}}{8}) - (\frac{-\pi}{3} + \frac{\sqrt{3}}{8})]$

$= \frac{1}{2}[(\ \frac{\pi}{3} - \frac{\sqrt{3}}{8} + \frac{\pi}{3} - \frac{\sqrt{3}}{8})]$

$= \frac{1}{2}[(\ \frac{2\pi}{3} - \frac{2\sqrt{3}}{8})]$

$= \ \frac{\pi}{3} - \frac{\sqrt{3}}{8}$